Application of Derivative

① Intermediate Value theorem :-

Let $f(x)$ be a Continuous function in the interval (α, β) and $a, b \in [\alpha, \beta]$ Satisfying $f(a) \cdot f(b) < 0$, then $\exists$ at least one point $c \in (a, b)$ for which $f(c) = 0$

② Lagranges Mean Value theorem :-

Let $f(x)$ be a function defined in $[a, b]$ Satisfying

① $f(x)$ is Continuous in $[a, b]$

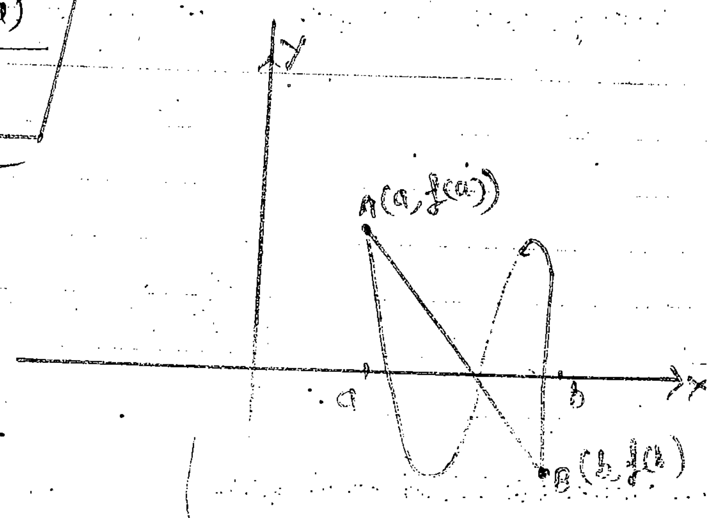
② $f(x)$ is differentiable in (a, b)

then $\exists$ at least one $c \in (a, b)$ such that

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$

Geometrical Interprotaction -

$$\text{grad } AB = \frac{f(b) - f(a)}{b - a}$$



$f'(c) = f'(x)$ at $x=c$.

$= $ Slope of tangent at $x=c$

i.e. ~~Slope of tangent~~ at $x=c$

$= $ grad AB

hence tangent at $x=c$ will be parallel to the chord joining AB.

i.e. If the curve represented by $y=f(x)$ is without breaking and smooth in the interval (a, b).

then $\exists$ a tangent to the curve at $x=c$ ~~with~~ which is parallel to the chord joining the point A and B.

③ Roll's theorems:-

Let $f(x)$ be a function defind in $[a, b]$ satisfying -

① $f(x)$ is continuous in $[a, b]$

② $f(x)$ is differentiable in (a, b) and

③ $f(a) = f(b)$

then $\exists$ a point $c \in (a, b)$ such that $f'(c) = 0$

Note- Rolles theorem से Lagrange mean Valui theorem का explane होता है न कि Lagrange mean Value से Rolles theorem

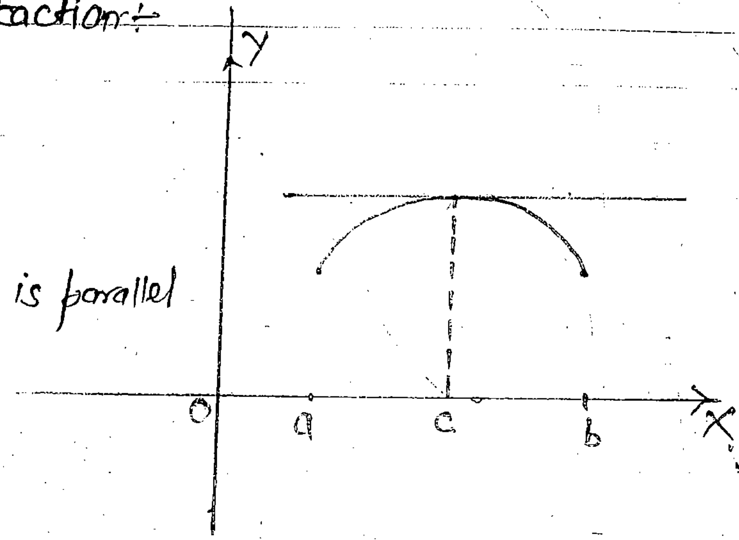
i.e. Rolles theorem $\Rightarrow$ Lagrange mean Value theorem.

Geometrical Interpretaction:-

$f'(c) = 0$

$\Rightarrow f'(x) = 0$ at $x = c$

at $x = c$ the tangent is parallel to x- axis



i.e. If the Curve is Smooth and Continuous and having the Same level at a, & b then $\exists$ at least one poin- c $\forall$ (a, b) the tangent at which will be parallel to x- axis.

Remark -1 In algebric point of View we Can write that eq^n $f'(x) = 0$ has at least one root in the interva (a, b) if function $f(x)$ Satisfing all the Conditions of Roll's theorem, infact with the help of equation $f'(x)$ we will find $f(x)$ (either intigrating or any other method) and we apply the condition of Roll's theorem on the fun=

(2) For the Continuty of the function we known that all polynomial function, exponential, trignometrical, logn etc are Continuous in their respected domain and for differentiability of the function we find $f'(x)$ and then absurve wather $f'(x)$ exist in the given interval or not

Ques- Find which of the following function Lagrange Mean Value theorem holds

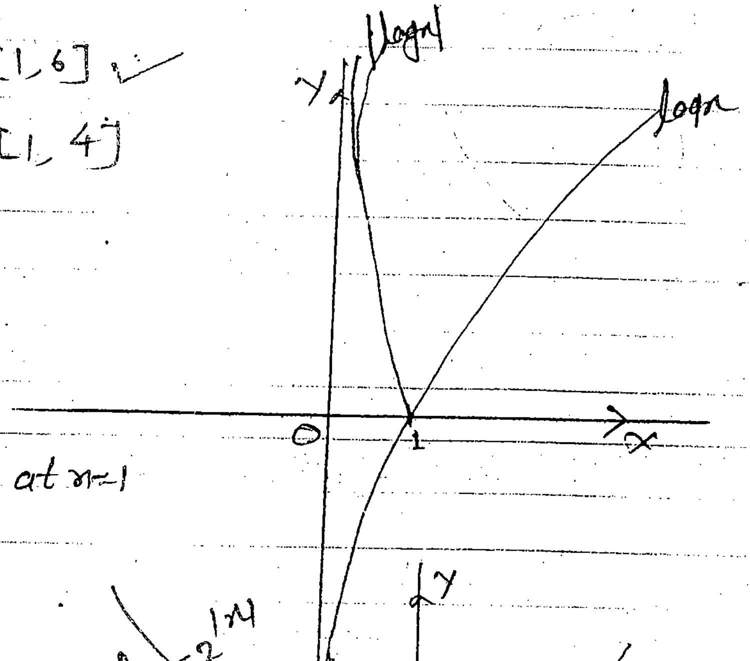
ⓐ $f(x) = |\log x|$ in $(0, 3)$

ⓑ $f(x) = 2^{|x|}$ in $[-1, 2]$

ⓒ $f(x) = |x^2+x+2|$ in $[1, 6]$ ✓

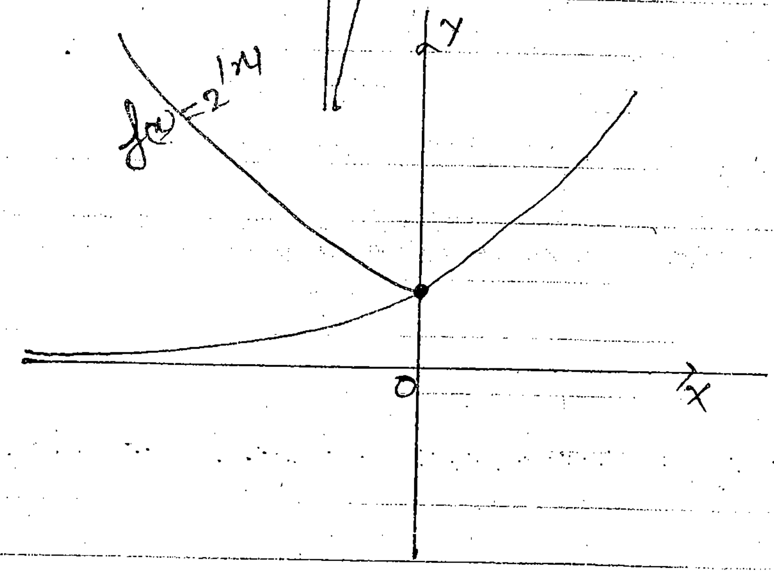
ⓓ $f(x) = \frac{1}{x-3}$ in $[1, 4]$

Sol ⓐ $f(x) = |\log x|$



function is not differentiable at $x=1$

ⓑ $f(x) = 2^{|x|}$



ⓒ $f(x) = |x^2+x+2|$ in $[1, 6]$

Since $f(x)$ is polynomial. hence it is continuous and differentiable

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Ques- Find the Value of c with the help of Lagrange mean Value theorem for the function $f(x) = x^2-4x+8$ in the interval [1,4]

Sol Since $f(x)$ is polynomial. hence it is Continuous and differentiable

$$f(x) = x^2-4x+8$$

$$f'(x) = 2x-4 \quad \text{also defined } (1,4)$$

i.e. $f(x)$ is differentiable in $(1,4)$

$$\text{So} \quad f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$2c-4 = \frac{f(4)-f(1)}{4-1}$$

$$= \frac{(16-16+8)-(1-4+8)}{3}$$

$$2c-4 = \frac{\cancel{3}}{\cancel{3}} = 1$$

$$2c = 5$$

$$c = \frac{5}{2} \quad \text{Ans} \quad c = \frac{5}{2}$$

Ques Find the point on the curve $y = x^2-4x+3$ tangent at which will be parallel to the chord forming point $x=2$ & $x=4$

Sol Since $f(x) = x^2-4x+3$ is polynomial. hence it is Continuous and differentiable in interval $(2,4)$

$$f(x) = x^2-4x+3$$

$$f'(x) = 2x-4$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$2c-4 = \frac{(\cancel{16}-\cancel{16}+3)-(4-8+3)}{4-2}$$

$$2c-4 = \frac{3+1}{2} = \frac{\cancel{4}\,2}{\cancel{2}}$$

$$2c = 6$$

$$c = 3$$

$\therefore$ Point $(c, f(c)) = (3, 0)$ Ans

Ques. If $2a+3b+6c=0$ then eqn $ax^2+bx+c=0$ has at least one root in

(a) $(0,2)$ (b) $(1,2)$ (c) $(0,1)$ (d) NOT.

Sol $f'(x) = ax^2+bx+c$

intigrating

$$f(x) = \frac{ax^3}{3}+\frac{bx^2}{2}+cx$$

So $f(x)$ is polynomial. hence it is continuous and differentiable in R.

$$f(a) = f(b)$$

$$f(0) = 0$$

$$f(2) = \frac{8a}{3}+2b+2c$$

$$f(1) = \frac{a}{3}+\frac{b}{2}+c$$

$$f(1) = \frac{2a+3b+6c}{6}$$

$$f(1) = 0 \qquad (\because 2a+3b+6c=0)$$

hence root $(0,1)$ Ans

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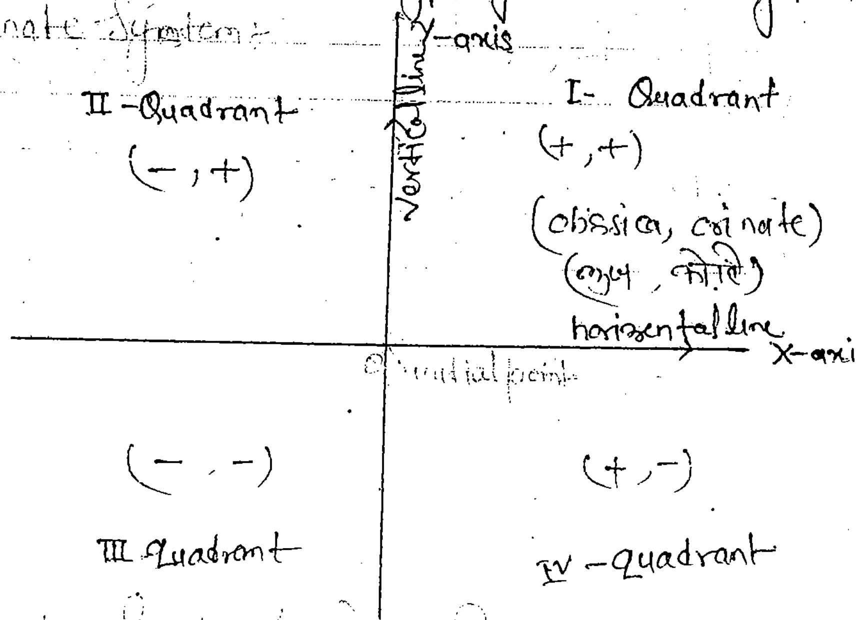
(1)

Coordinate Geometry

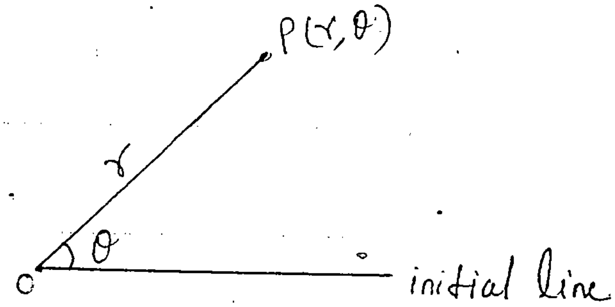
Coordinate System-

There are two type of Coordinate System

[1-] Cortision Coordinate System-



[2-] Polar Coordinate System (ध्रुवीय पद्धति)

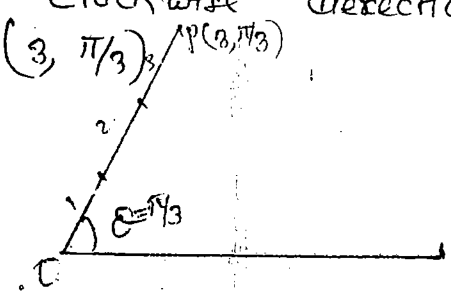


कोण नापने के नियम- कोण नापने के दो नियम है-

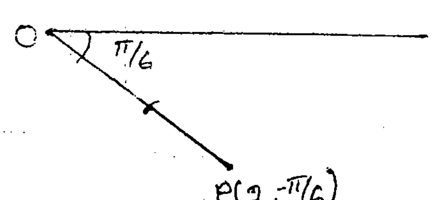
(I) Anticlockwise derection

(II) Clockwise derection

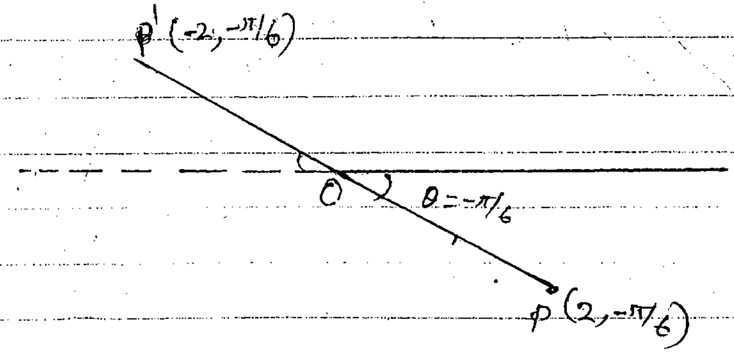
(I) $(3, \pi/3)$



(II) $(2, -\pi/6)$

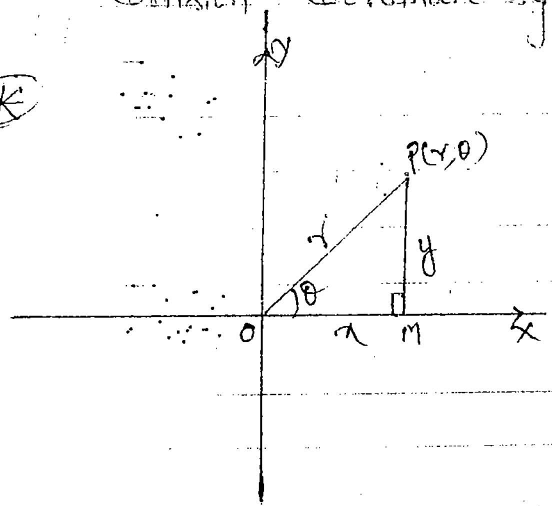


(iii) $(-2, -\pi/6)$



Note – Polar Coordinate System always Single Variable and Cartision Coordinate System always double Variable.

(*)



from ΔOPM –

$$\frac{x}{r} = \cos\theta$$

$$\Rightarrow \boxed{x = r\cos\theta} \text{ —— (I)}$$

and $$\frac{y}{r} = \sin\theta$$

$$\Rightarrow \boxed{y = r\sin\theta} \text{ —— (II)}$$

Squaring and adding

$$x^2 + y^2 = r^2(\sin^2\theta + \cos^2\theta)$$

$$\boxed{x^2 + y^2 = r^2}$$

and dividing (II) by (I)

$$y/x = \frac{r\sin\theta}{r\cos\theta} = \tan\theta \quad \Rightarrow \boxed{\tan\theta = y/x}$$

Que The Curve $r=a$ represent ----

(a) Circle ✓ (b) ellipse (c) St. line (d) NOT.

Sol

$r=a$

we known that

$r=\sqrt{x^2+y^2}$ $\Rightarrow \sqrt{x^2+y^2}=a$

$x^2+y^2=a^2$

represent a Circle whose Centre $(0,0)$ and radius a.

Que Curve $r=2\sec\theta$ represent

(a) St. line ✓ (b) Circle (c) ellipse (d) NOT.

Sol Curve $r=2\sec\theta$

$r=2\cdot\frac{1}{\cos\theta}$ $\Rightarrow r\cos\theta=2$ (Since $x=r\cos\theta$

$x=2$

represent a St. line

Que $r=a\sin(\theta-\alpha)$ represent a

(a) ellipse (b) parabala (c) Circle (d) Hyperbola

Sol

$r=a\sin(\theta-\alpha)$

$r=a(\sin\theta\cdot\cos\alpha-\cos\theta\sin\alpha)$

$r^2=ar(\sin\theta\cos\alpha-\cos\theta\sin\alpha)$

$r^2=a\cos\alpha\cdot r\sin\theta-a\sin\alpha\cdot r\cos\theta$

$x^2+y^2=(a\cos\alpha)y-(a\sin\alpha)x$

$\Rightarrow x^2+y^2+a\sin\alpha\, x-a\cos\alpha\cdot y=0$

represent a circle

Que- The Curve $r=4\operatorname{cosec}\theta$ represent.

Sol- $r=4\operatorname{cosec}\theta$

$r=4\cdot\frac{1}{\sin\theta}$ $\Rightarrow r\sin\theta=4$

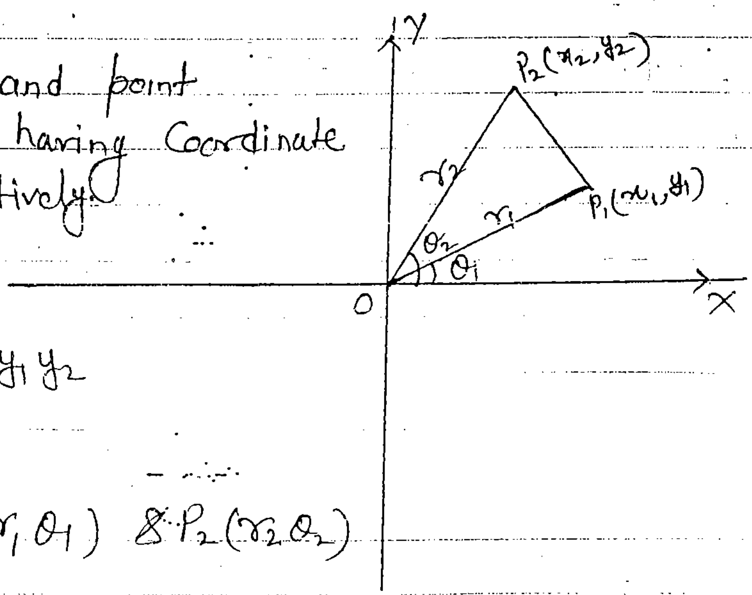
$\boxed{y=4}$ represent a S.t. line

Que) $r = 5\cos\theta$ represent

$r = 5\cos\theta$

$r^2 = 5^2\cos^2\theta$

$x^2+y^2 = 25\cos\theta$ represent a circle.



(*) Let O be the pole and point P_1, P_2 be two point having Coordinate (x_1,y_1) & (x_2,y_2) respectively. then ~~prope~~ that

$$OP_1 \cdot OP_2 \cos P_1OP_2 = x_1x_2 + y_1y_2$$

From figer: Suppose $P_1(r_1,\theta_1)$ & $P_2(r_2,\theta_2)$

and Consider $OP_1 \cdot OP_2 \cos P_1OP_2$

$$= r_1 \cdot r_2 \cos(\theta_2-\theta_1)$$

$$\cancel{= r_1 r_2 \cos\theta_2(\cos\theta_1 + \sin\theta_2\sin\theta_1)}$$

$$= (r_1\cos\theta_1)(r_2\cos\theta_2) + (r_1\sin\theta_1)(r_2\sin\theta_2)$$

$$= x_1x_2 + y_1y_2$$

$$\therefore \boxed{OP_1 \cdot OP_2 \cos P_1OP_2 = x_1x_2+y_1y_2}$$

(*) Distance b/w P_1 & P_2 :-
from above figer

$$P_1 \cdot P_2 = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$P_1 \cdot P_2 = \sqrt{(r_2\cos\theta_2 - r_1\cos\theta_1)^2 + (r_2\sin\theta_2 - r_1\sin\theta_1)^2}$$

$$P_1 \cdot P_2 = \sqrt{r_2^2 \cos^2\theta_2 + r_1^2 \cos^2\theta_1 - 2r_1r_2 \cos\theta_1 \cdot \cos\theta_2 + r_2^2 \sin^2\theta_2 + r_1^2 \sin^2\theta_1 - 2r_1r_2 \sin\theta_1 \sin\theta_2}$$

$$= \sqrt{r_2^2(\cos^2\theta_2 + \sin^2\theta_2) + r_1(\cos^2\theta_1 + \sin^2\theta_1) - 2r_1r_2(\cos\theta_1 \cdot \cos\theta_2 + \sin\theta_1 \sin\theta_2)}$$

$$P_1 \cdot P_2 = \sqrt{r_2^2 + r_1^2 - 2r_1r_2 \cos(\theta_2 - \theta_1)}$$

$$\boxed{P_1P_2^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_2 - \theta_1)}$$

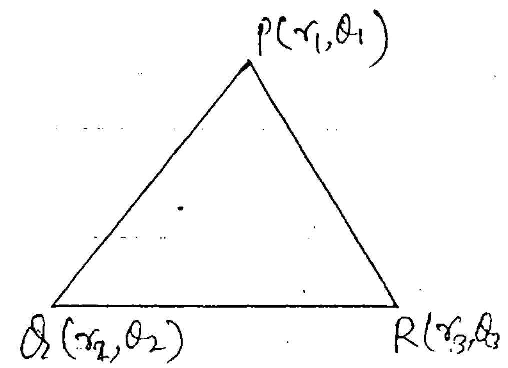
Other method –

$$\cos P_1OP_2 = \frac{OP_1^2 + OP_2^2 - P_1P_2^2}{2\, OP_1 \cdot OP_2}$$

$$\cos(\theta_2 - \theta_1) = \frac{r_1^2 + r_2^2 - (P_1P_2)^2}{2r_1r_2}$$

$$(P_1P_2)^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_2 - \theta_1)$$

* Area of ΔPQR –



Suppose ΔPQR be the tringle whose coordinate $(r_1 \theta_1)$ $(r_2 \theta_2)$ & $(r_3 \theta_3)$ respectivily.

Area of ΔPQR

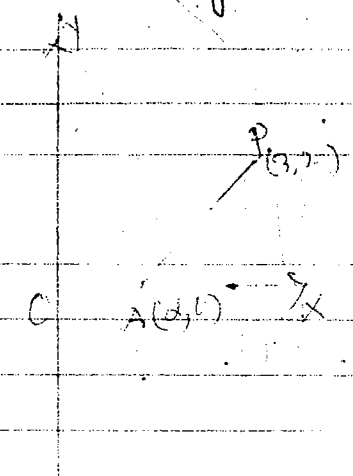
$$= \frac{1}{2}[r_2r_3 \sin(\theta_3 - \theta_2) + r_3r_1 \sin(\theta_1 - \theta_2) + r_1r_2 \sin(\theta_2 - \theta_1)$$

Ques Find the point on x-axis whose distance from the point (3,2) is 3-Unit.

Sol

Given that

$AP = 3$



$AP = \sqrt{(\alpha-3)^2+(0-2)^2} = 3$

$3^2 = (\alpha-3)^2+(-2)^2$

$\cancel{9} = \alpha^2+\cancel{9}-6\alpha+4$

$\Rightarrow \alpha^2-6\alpha+4=0$

$\therefore \alpha = \dfrac{+6 \pm \sqrt{(6)^2-4\cdot1\cdot4}}{2\cdot1}$

$= \dfrac{6 \pm \sqrt{36-16}}{2}$

$= \dfrac{6 \pm \sqrt{20}}{2} = \dfrac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$

point $(3\pm\sqrt{5}, 0)$ **Ans**

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29/10/015

⊛ Exact differential eqⁿ :-

The differential equations $Mdx + Ndy = 0$ is said to be exact if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

this is known as Condition of exactness.

For Solution:

$$\int M dx + \int (\text{those term of } N \text{ which are free form } x) dy$$

Ques $(6x^2y + 3xy^2)dx + (3x^2y + 2x^3)dy = 0$

$$\frac{\partial M}{\partial y} = 6x^2 + 6xy$$

$$\frac{\partial N}{\partial x} = 6xy + 6x^2 = 6x^2 + 6xy$$

$$\therefore \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$ So equation becomes exact.

hence sol. $\int (6x^2y + 3xy^2) dx + \int 0 \, dy$

$$= \cancel{6}^{2} \frac{x^3}{\cancel{3}} y + \frac{3x^2y^2}{2} = 2x^3y + \frac{3}{2}x^2y^2 \quad \text{Ans}$$

Ques $(e^y + 1)\cos x \, dx + (e^y \sin x \, dy = 0$

$$\frac{\partial M}{\partial y} = e^y \cos x \quad \text{and} \quad \frac{\partial N}{\partial x} = e^y \cos x$$

$$\therefore \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$ hence equation becomes exact.

hence Solution:

$$\int (e^y + 1)\cos x \, dx$$

$$= (e^y + 1)\sin x + C \quad \text{Ans}$$

Ques- $x\,dx + y\,dy = \frac{a^2(x\,dy - y\,dx)}{x^2+y^2}$

Sol- $\left(x + \frac{a^2y}{x^2+y^2}\right)dx + \left(y - \frac{a^2x}{x^2+y^2}\right)dy = 0$

$$\frac{\partial M}{\partial y} = \frac{(x^2+y^2)\frac{d}{dy}a^2y - a^2y\frac{d}{dy}(x^2+y^2)}{(x^2+y^2)^2}$$

$$= \frac{(x^2+y^2)a^2 - a^2y(2y)}{(x^2+y^2)^2} = \frac{a^2(x^2+y^2-2y^2)}{(x^2+y^2)^2} = \frac{a^2(x^2-y^2)}{(x^2+y^2)^2}$$

$$\frac{\partial N}{\partial x} = \frac{-\{(x^2+y^2)a^2 - a^2x\cdot 2x\}}{(x^2+y^2)^2}$$

$$= \frac{-a^2\{x^2+y^2-2x^2\}}{(x^2+y^2)^2} = \frac{a^2(x^2-y^2)}{(x^2+y^2)^2}$$

$\therefore \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ So equation becomes exact

hence Solution $\int\left(x + \frac{a^2y}{x^2+y^2}\right)dx + \int y\,dy$

$$= \frac{x^2}{2} + a^2y\cdot\frac{1}{y}\tan^{-1}\left(\frac{x}{y}\right) + \frac{y^2}{2}$$

$$= \frac{x^2}{2} + \frac{y^2}{2} + a^2\tan^{-1}(x/y) + c \quad \underline{\text{Ans}}$$

Ques $(x^2-ay)dx = (ax-y^2)dy$

$(x^2-ay)dx + (y^2-ax)dy = 0$

$\frac{\partial M}{\partial y} = -a$ & $\frac{\partial N}{\partial x} = -a$

$\therefore \boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ So eq^n becomes exact

hence Solution $\int(x^2-ay)dx + \int y^2\,dy$

$$= \frac{x^3}{3} + \frac{y^3}{3} - axy + C \quad \underline{\text{Ans}}$$

(*) If this equation is not exact then we used intigrating factor.

⟨i⟩ If $\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=f(x)$ then

intigrating factor $e^{\int f(x)\,dx}$

⟨ii⟩ If $\frac{1}{M}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=f(y)$ then

intigrating factor $e^{-\int f(y)\,dy}$

⟨iii⟩ If equation can be written as $f_1(x,y)y\,dx+f_2(x,y)x\,dy$

I.F. $\frac{1}{Mx-Ny}$ provided $Mx-Ny\neq 0$

(iv) If equation can be written as $x^{\alpha}y^{\beta}(my\,dx+nx\,dy)$
$+x^{\gamma}y^{\delta}(py\,dx+qx\,dy)=0$

then we will assume intigrating factor as $x^h.y^k$ an after multiplication and using condition of exactness we will get two eqn h, & K and hence (h, K)

Ques- $y\,dx-x\,dy+(1+x^2)\,dx+x^2\sin y\,dy=0$

$\Rightarrow (x^2+y+1)\,dx+(x^2\sin y-x)\,dy=0$

$\frac{\partial M}{\partial y}=1$ & $\frac{\partial N}{\partial x}=2x\sin y-1$

$\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)=1-2x\sin y+1=-2(x\sin y-1)$

$\frac{1}{N}\cdot -2(x\sin y-1)=-\frac{2}{x}\frac{\cancel{(x^2\sin y-x)}}{\cancel{(x^2\sin y-x)}}=-\frac{2}{x}=f(x)$

hence I.F. $e^{\int f(x)\,dx}=e^{-\int\frac{2}{x}dx}$
$=e^{-2\log x}$
$=e^{-2}$ Ans

Ques $y(axy+e^x)dx - e^x dy = 0$

$$\frac{\partial M}{\partial y} = 2axy + e^x$$

$$\frac{\partial N}{\partial x} = -e^x$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2axy + 2e^x = \frac{2}{y}y(axy+e^x)$$

$$\therefore \frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{\cancel{y(axy+e^x)}} \cdot \frac{2}{y}\,\cancel{y(axy+e^x)} = \frac{2}{y} = f(y)$$

then I.F. $= e^{-\int f(y)} = e^{-\int \frac{2}{y}}$

$$= e^{-2\log y}$$

$$= e^{-2} \text{ Ans}$$

Ques $(y^2 - 2x^2y)dx + (2x^3 - xy)dy = 0$

$$(y - 2x^2)y\,dx + (2x^2 - y)x\,dy = 0$$

$$\therefore \frac{\partial M}{\partial y} = 2y - 2x^2 \quad \& \quad \frac{\partial N}{\partial x} = (6x^2 - y)$$

Hence Not exact

$\therefore$ Hence I.F $= \int \frac{1}{Mx - Ny}$

$$= \int \frac{1}{(y^2+2x^2y)x - (2x^3 - xy)y}$$

$$= \int \frac{1}{xy^2 + \cancel{2x^3y} - \cancel{2x^3y} + xy^2}$$

$$= \int \frac{1}{2xy^2} = \frac{1}{2}\left[\log(xy^2)\right]$$

$$= \frac{1}{2}\log x + \frac{1}{2}\log y^2$$

$$= \frac{1}{2}\log x + \log y \text{ Ans}$$

Ques - If the order of the differential eqⁿ whose general eqⁿ is given by

$$y = (c_1+c_2)\cos(x+c_3) - c_4 e^{x+c_5}$$

where c_1, c_2, c_3, c_4 & c_5 are arbitrary Constant is

(a) 5 (b) 4 (c) 3 (d) 2

Sol General eqⁿ

$$y = (c_1+c_2)\cos(x+c_3) - c_4 e^{x+c_5}$$

$$= A_1 \cos(x+c_3) - c_4 e^{x} \cdot e^{c_5}$$

$$= A_1 \cos(x+c_3) - A_2 e^{x}$$

$$= \underline{A_1} \cos(x+\underline{A_3}) - \underline{A_2} e^{x}$$

Ques The diffⁿ eqⁿ representing the faimly of Curve $y^2 = 2c(x+\sqrt{c})$ where c is a parameter is of

(a) order 1 (b) order 2 (c) degree 3 (d) degree 4

Sol Eqⁿ of Curve

$$y^2 = 2c(x+\sqrt{c}) \quad \text{--- (i)}$$

$$2y\frac{dy}{dx} = 2c \quad \text{--- (ii)}$$

Substitute these value in (i)

$$y^2 = 2y\frac{dy}{dx}\left(x + \sqrt{y\frac{dy}{dx}}\right)$$

$$y^2 = 2xy\frac{dy}{dx} + \sqrt{\left(y\frac{dy}{dx}\right)} \cdot 2y\frac{dy}{dx}$$

hence order = 1 degree = 3

Ques $(1+t)\frac{dy}{dx} - ty = 1$ and $y(0) = -1$ then $y(1)$ is equal to

(a) $-\frac{1}{2}$ (b) $e+\frac{1}{2}$ (c) $e-\frac{1}{2}$ (d) $\frac{1}{2}$

Sol $(1+t)\frac{dy}{dt} - ty = 1$

$\frac{dy}{dt} + t\frac{dy}{dt} - ty = 1$ $\quad P = -\frac{t}{1+t} \quad Q = \frac{1}{1+t}$

$\frac{dy}{dt} - \frac{t}{1+t} = \frac{1}{1+t}$

$I.F. = e^{\int -\frac{t}{1+t}dt} = e^{-\int(1-\frac{1}{1+t})dt}$

$= e^{-\{t - \log(1+t)\}}$

$= e^{-t} \cdot e^{\log(1+t)}$

$= e^{-t}(1+t)$

hence solution of eqn

$y \cdot e^{-t}(1+t) = \int \frac{1}{\cancel{(1+t)}} e^{-t}\cancel{(1+t)}\,dt + c$

$y\, e^{-t}(1+t) = e^{-t} + c$

given $y(0) = -1$

we put $t = 0 \quad y = -1$

$(-1)\, e^{0}(1+0) = -e^{-0} + c$

$\cancel{-1} = \cancel{-1} + c$

$\boxed{c = 0}$

$\therefore\ y e^{-t}(1+t) = -e^{-t}$

Value of y: $y\, e^{-1}(1+1) = -e^{-1}$

$\boxed{y = -1/2}$ Ans

Ques - $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x$ given $y(0) = 1$ then $y(\frac{\pi}{2})$ is equal to

(a) $1/3$ (b) $2/3$ (c) $-1/3$ (d) 1

Ques- Find Area of Tringle with Vertices $A(1,1,2)$ $B(2,3,5)$ & $C(1,5,5)$

$\overline{AB} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overline{AC} = 0\hat{i} + 4\hat{j} + 3\hat{k}$

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 0 & 4 & 3 \end{vmatrix}$$

$$= \hat{i}(6-12) - \hat{j}(3-0) + \hat{k}(4-0)$$

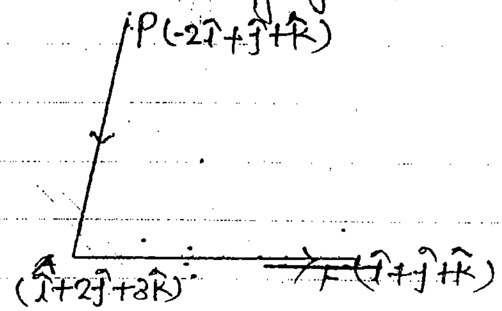
$$= -6\hat{i} - 3\hat{j} + 4\hat{k}$$

Area of $\Delta ABC = \frac{1}{2}|\overline{AB} \times \overline{AC}|$

$$= \frac{1}{2}\sqrt{(-6)^2 + (-3)^2 + 4^2}$$

$$= \frac{1}{2}\sqrt{36+9+16} = \frac{1}{2}\sqrt{61} \text{ Unit } \underline{\text{Ans}}$$

Ques- The force $\hat{i}+\hat{j}+\hat{k}$ passes through a point $-2\hat{i}+\hat{j}+\hat{k}$ then find moment of force about the point $\hat{i}+2\hat{j}+3\hat{k}$



$F = \hat{i}+\hat{j}+\hat{k}$

$\overline{AP} = r = -3\hat{i} - \hat{j} - 2\hat{k}$

Moment of force $= |r \times F|$

$$= \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -1 & -2 \\ 1 & 1 & 1 \end{vmatrix} \right|$$

$$= |\hat{i}(-1+2) - \hat{j}(-3+2) + \hat{k}(-3+1)|$$

$$= |\hat{i} + \hat{j} - 2\hat{k}|$$

$$= \sqrt{1^2 + 1^2 + (-2)^2}$$

$$= \sqrt{1+1+4} = \sqrt{6} \text{ Unit } \underline{\text{Ans}}$$

Ques A Force of Magnitude 6 Unit is acting along the direction $\overline{AB}$ where $A(2,1,0)$ $B(3,-1,2)$ Find the moment of this forces about origin.

Sol $\overline{AB} = \hat{i}-2\hat{j}+2\hat{k}$ Unit vector $= \dfrac{\overline{AB}}{|\overline{AB}|}$

$$\text{Unit vector } \overline{AB} = \frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{1^2+(-2)^2+2^2}}$$

$$= \frac{\hat{i}-2\hat{j}+2\hat{k}}{\sqrt{1+4+4}} = \frac{1}{3}(\hat{i}-2\hat{j}+2\hat{k})$$

$$\text{Force } \overline{F} = \cancel{6}^{2} \times \frac{1}{\cancel{3}}(\hat{i}-2\hat{j}+2\hat{k})$$

$$= 2\hat{i}-4\hat{j}+4\hat{k}$$

Suppose origin $O(0\hat{i}+0\hat{j}+0\hat{k})$

$$\overline{OA} = \overline{r} = 2\hat{i}+\hat{j}-0\hat{k}$$

$$\text{Moment } F = |\overline{r}\times F|$$

$$= \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 0 \\ 2 & -4 & 4 \end{vmatrix} \right|$$

$$= |\hat{i}(4-0)-\hat{j}(8-0)+\hat{k}(-8-2)|$$

$$= |4\hat{i}-8\hat{j}-10\hat{k}|$$

$$= \sqrt{16+64+100}$$

$$= \sqrt{180}$$

$$= 3\sqrt{20} \text{ Unit}$$ Ans

Ques- Find Value of $\hat{i}\times(\bar{a}\times\hat{i})+\hat{j}\times(a\times\hat{j})+\hat{k}\times(a\times\hat{k})$.

Sol $\hat{i}\times(a\times\hat{i})$

$$= (\hat{i}\cdot\hat{i})\bar{a}-(\hat{i}\cdot\bar{a})\cdot\hat{i}$$

$$= 1\cdot\bar{a} - a_1\hat{i}$$

Similarly $\hat{j}\times(\bar{a}\times\hat{j}) = (\hat{j}\cdot\hat{j})\bar{a} - (\hat{j}\cdot\bar{a})\cdot\hat{j}$

$= \bar{a} - a_2\hat{j}$

and $\hat{k}\times(\bar{a}\times\hat{k}) = (\hat{k}\cdot\hat{k})\cdot\bar{a} - (\hat{k}\cdot\bar{a})\cdot\hat{k}$

$= \bar{a} - a_3\hat{k}$

$\therefore\ \hat{i}\times(\bar{a}\times\hat{i}) + \hat{j}\times(\bar{a}\times\hat{j}) + \hat{k}\times(a\times\hat{k})$

$= \bar{a} - a_1\hat{i} + \bar{a} - a_2\hat{j} + \bar{a} - \bar{a}_3\hat{k}$

$= 3\bar{a} - (\bar{a}_1\hat{i} + \bar{a}_2\hat{j} + \bar{a}_3\hat{k})$

$= 3\bar{a} - \bar{a}$

$= 2\bar{a}$ Ans

Ques- The value of $[\bar{a}\times\bar{b},\ \bar{b}\times\bar{c},\ \bar{c}\times\bar{a}]$ is equal to

Sol Since $[\bar{a}\ \bar{b}\ \bar{c}] = \bar{a}\cdot(\bar{b}\times\bar{c})$

$[\bar{a}\times\bar{b},\ \bar{b}\times\bar{c},\ \bar{c}\times\bar{a}]$

$= (\bar{a}\times\bar{b})\cdot\{(\bar{b}\times\bar{c})\times(\bar{c}\times\bar{a})\}$

$= (\bar{a}\times\bar{b})\{[\bar{b}\ \bar{c}, \bar{c}]\bar{a} - [\bar{b}\ \bar{a}\ \bar{c}]\bar{c}\}$

$= (\bar{a}\times\bar{b})\{[\bar{a}\ \bar{b}\ \bar{c}]\cdot\bar{c}\}$

$= [\bar{a}\ \bar{b}\ \bar{c}]\ (\bar{a}\times\bar{b})\cdot\bar{c}$

$= [\bar{a}\ \bar{b}\ \bar{c}]\ [\bar{a}\ \bar{b}\ \bar{c}]$

$= [\bar{a}\ \bar{b}\ \bar{c}]^2$ Ans

Ques Find Value of $[\bar{a}+\bar{b},\ \bar{b}+\bar{c},\ \bar{c}+\bar{a}]$

Since $[\bar{a}, \bar{b}\bar{c}] = \bar{a}\cdot(\bar{b}\times\bar{c})$

$[\bar{a}+\bar{b},\ \bar{b}+\bar{c},\ \bar{c}+\bar{a}]$

$= (\bar{a}+\bar{b})\cdot\{(\bar{b}+\bar{c})\times(\bar{c}+\bar{a})\}$

$= (\bar{a}+\bar{b})\{\bar{b}\times(\bar{c}+\bar{a}) + \bar{c}\times(\bar{c}+\bar{a})\}$

$= (\bar{a}+\bar{b})\{\bar{b}\times\bar{c} + \bar{b}\times\bar{a} + \bar{c}\times\bar{c} + \bar{c}\times\bar{a}\}$

$= (\bar{a}+\bar{b})\{\bar{b}\times\bar{c} + \bar{b}\times\bar{a} + \bar{c}\times\bar{a}\}$

$= \bar{a}\cdot(\bar{b}\times\bar{c}) + \bar{a}(\bar{b}\times\bar{a}) + \bar{a}\cdot(\bar{c}\times\bar{a}) + \bar{b}\cdot(\bar{b}\times\bar{c}) + \bar{b}\cdot(\bar{b}\times\bar{a}) + \bar{b}(\bar{c}\times\bar{a})$

$= [\bar{a}\ \bar{b}\ \bar{c}] + [\bar{a}\ \bar{b}\ \bar{a}] + [\bar{a}\ \bar{c}\ \bar{a}] + [\bar{b}\ \bar{b}\ \bar{c}] + [\bar{b}\ \bar{b}, \bar{a}] + [\bar{b}\ \bar{c}\ \bar{a}]$

$= [\bar{a}\ \bar{b}\ \bar{c}] + 0 + 0 + 0 + 0 + [\bar{a}\ \bar{b}\ \bar{c}]$

$= 2[\bar{a}\bar{b}\bar{c}]$ Ans

Ques- Prove that the four point having position vector A(4, 5, 1) B (0, -1, -1), C(3, 9, 4) and D(-4, 4, 4) are Coplaner

Sol

$\overline{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k}$

$\overline{AC} = -\hat{i} + 4\hat{j} + 3\hat{k}$

$\overline{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$

Since $\overline{AB}, \overline{AC}, \overline{AD}$ are Coplaner

$[\overline{AB}, \overline{AC}, \overline{AD}] = 0$

$$\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$

$= -4(12+3) + 6(-3+24) - 2(1+32)$

$= -60 + 126 - 60$

$= 0$ Hence proved

Ques- Find Value of λ For which the vector $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are Coplaneor.

Sol

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -1 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

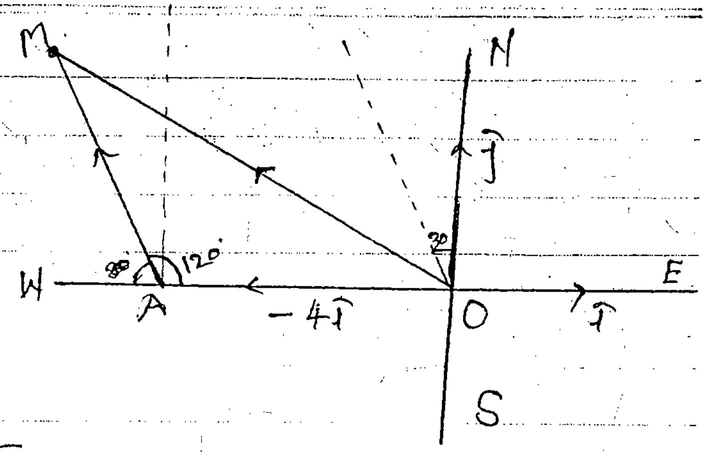
$2(10+\lambda) + 1(5+3) + 1(\lambda - 6) = 0$

$20 + 2\lambda + 8 + \lambda - 6 = 0$

$3\lambda + 22 = 0$

$\lambda = -\frac{22}{3}$ Ans

Ques- A person moves 4 km in west direction after that he moves 3 Km. in the direction of 30° west of North then find the displacement of person from the Starting points.



Unit vector

$\overline{AM} = 3(\hat{i}\cos 120° + \hat{j}\sin 120°)$

$= 3(\hat{i}(-\sin 30°) + \hat{j}\cos 30°)$

$= 3\left\{\hat{i}\left(\frac{-1}{2}\right) + \hat{j}\left(\frac{\sqrt{3}}{2}\right)\right\}$

$= -\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

Displacement $\overline{OM} = \overline{OA} + \overline{AM}$

$= -4\hat{i} + \left(-\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$

$= \left(-4 - \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

$= -\frac{11}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

Ques- If $\bar{a}, \bar{b}, \bar{c}$ are unit vector such that $\bar{a}+\bar{b}+\bar{c}=0$ then find value $\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a}$.

Sol - $\because \bar{a}+\bar{b}+\bar{c}=0$

$\therefore |\bar{a}+\bar{b}+\bar{c}| = 0$

$|\bar{a}+\bar{b}+\bar{c}|^2 = 0$

$(\bar{a}+\bar{b}+\bar{c})(\bar{a}+\bar{b}+\bar{c}) = 0$

$\bar{a}\cdot\bar{a} + \bar{a}\cdot\bar{b} + \bar{a}\bar{c} + \bar{b}\bar{a} + \bar{b}\bar{b} + \bar{b}\bar{c} + \bar{c}\bar{a} + \bar{c}\bar{b} + \bar{c}\bar{c} = 0$

$|\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a}\bar{b} + \bar{b}\bar{c} + \bar{c}\bar{a}) = 0$

$1 + 1 + 1 + 2(\bar{a}\bar{b} + \bar{b}\bar{c} + \bar{c}\bar{a}) = 0$

$\bar{a}\bar{b} + \bar{b}\bar{c} + \bar{c}\bar{a} = -\frac{3}{2}$. Ans

Ques If $\bar{a}, \bar{b}, \bar{c}$ are three unit vector and each one of them is perpendicular to the sum of other to then find value of $|\bar{a}+\bar{b}+\bar{c}|$

Sol $|a+b+c|^2$

$= (\bar{a}+\bar{b}+\bar{c})(\bar{a}+\bar{b}+\bar{c})$

$= \bar{a}\bar{a} + \bar{a}\cdot(\bar{b}+\bar{c}) + \bar{b}\cdot\bar{b} + b(\bar{a}+\bar{c}) + \bar{c}\cdot\bar{c} + c\cdot(\bar{a}+\bar{b})$

$= |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 0 + 0 + 0 + 0$

$= 1+1+1$

$= 3$

$\therefore |\bar{a}+\bar{b}+\bar{c}| = \sqrt{3}$

<u>Ques</u>- The $\bar{a}.\bar{b}\,\bar{c}$ are equally to magnitude and mutully perpendicular then find the angle b/w resultent vector $\bar{a}+\bar{b}+\bar{c}$ with $\bar{a}.\bar{b}\,\bar{c}$.

Sol-
$$\cos\theta = \frac{(a+b+c).\bar{a}}{|a+b+c|\,|a|}$$

Condition fo mutualy $\perp^r$ i.e. $\bar{a}.\bar{b}+\bar{b}.\bar{c}+\bar{c}.\bar{a}=0$

$$\cos\theta = \frac{\bar{a}.\bar{a}+\bar{a}.\bar{b}+\bar{a}.\bar{c}}{|\sqrt{3}K|\,|K|} = \frac{K^2}{\sqrt{3}K^2}$$

$$\cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \boxed{\theta = \cos^{-1}(1/\sqrt{3})}$$

Since
$|\bar{a}+\bar{b}+\bar{c}|^2 = |\bar{a}|^2+|\bar{b}|^2+|\bar{c}|^2$
$= K^2+K^2+K^2$
$= 3K^2$
$\therefore |\bar{a}+\bar{b}+\bar{c}| = \sqrt{3}K$

Similarly for $\bar{b}$ & $\bar{c}$

$$\boxed{\theta = \cos^{-1}(1/\sqrt{3})}$$

& $\boxed{\theta = \cos^{-1}(1/\sqrt{3})}$

<u>Ques</u> If the vecter of ΔABC has their position vector $\bar{a}\,\bar{b}\,\bar{c}$, then prove that area of Tringle $= \frac{1}{2}|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}|$

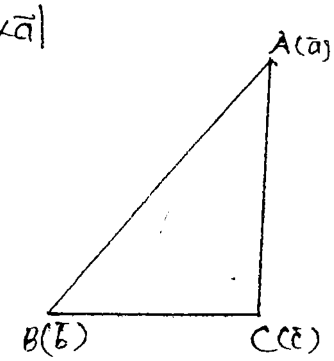
<u>Sol</u> We know that

Area of $\Delta = \frac{1}{2}|\overline{AB}\times\overline{AC}|$

$= \frac{1}{2}|(\bar{b}-\bar{a})\times(\bar{c}-\bar{a})|$

$= \frac{1}{2}|\bar{b}\times\bar{c}-\bar{b}\times\bar{a}-\bar{a}\times\bar{c}+\bar{a}\times\bar{a}|$

$= \frac{1}{2}|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}|$



Hence proved

(106)

18/06/018 Circle

(1)

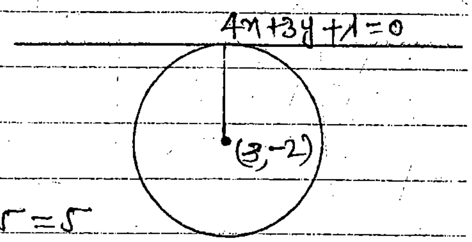
Ques Find eq^n to the tangent to the circle $x^2+y^2-6x+4y-12=0$ which are parallel and perpendicular to the line $4x+3y+5=0$

Sol eq^n of circle

$x^2+y^2-6x+4y-12=0$

Centre $=(3,-2)$

radius $=\sqrt{(3)^2+(-2)^2+12}=\sqrt{18+12}=\sqrt{25}=5$



length of perpendicular to the line $4x+3y+\lambda=0$

$$\left|\frac{4\cdot3+3\times(-2)+\lambda}{\sqrt{4^2+3^2}}\right|=5$$

$$\left|\frac{12-6+\lambda}{\sqrt{25}}\right|=5$$

$$\frac{6+\lambda}{5}=\pm5$$

$$6+\lambda=\pm25$$

$$\lambda=25-6=19$$

$$\text{and } \lambda=-25-6=-31$$

hence eq^n of tangent

$4x+3y+19=0$ **Ans**

or $4x+3y-31=0$

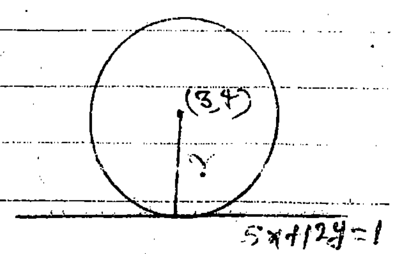
Ques- The equation of circle which touches the line $5x+12y=1$ and which has its centre at $(3,4)$ is

(a) $(x-3)^2+(y-2)^2=\left(\frac{62}{11}\right)^2$ (b) $(x-3)^2+(y-4)^2=\left(\frac{62}{11}\right)^2$

(c) $(x-3)^2+(y-4)^2=\left(\frac{62}{13}\right)^2$ ✓ (d) ~~$(x-3)^2+$~~ N.O.T.

Sol length of $\perp^n$ at point $(3,4)$

$$\left|\frac{5\cdot3+12\cdot4-1}{\sqrt{(5)^2+(12)^2}}\right|=r$$



$$r = \left|\frac{15+48-1}{\sqrt{25+144}}\right| = \left|\frac{62}{\sqrt{169}}\right| = \frac{62}{13}$$

Hence eq^n of Circle

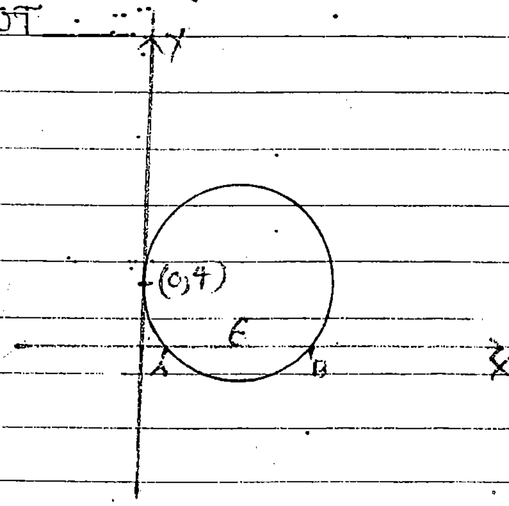
$$(x-3)^2+(y-4)^2 = \left(\frac{62}{13}\right)^2 \quad \underline{Ans}$$

Ques Find eq^n of Circle which touches the axis of y at the distance $+4$ from Origin and cuts of intercept of 6 unit on x-axis is

(a) $x^2+y^2-10x-8y+16=0$ (b) $x^2+y^2+10x-8y+16=0$

(c) $x^2+y^2-10x+8y+16=0$ (d) NOT

Sol Circle touches y-axis



$$(0,-f) = (0,4)$$

$$\boxed{f = -4}$$

and $f^2 = c$

$$c = 4^2 = 16$$

$$\boxed{c=16}$$

and Circle intercept on x-axes

$$2\sqrt{g^2-c} = 0$$

$$\sqrt{g^2-16} = 0$$

$$g^2-16 = 0$$

$$\boxed{g = \pm 4}$$

Hence eq^n of circle becomes

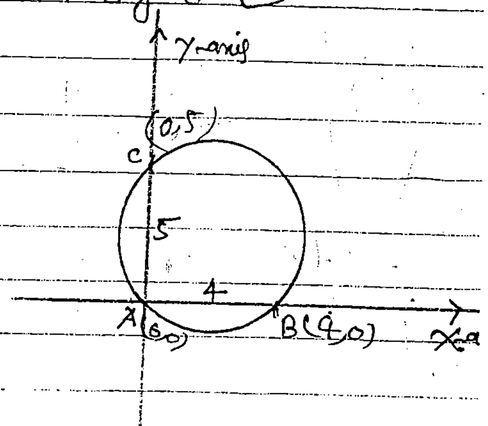
$$x^2+y^2+2gx+2fy+c=0$$

$$x^2+y^2-8x-8y+16=0 \quad \underline{Ans}$$

Ques- The equation of circle passes through the origin and making intercepts (4,5) on the coordinate axis is

(a) $x^2+y^2-4x+5y=0$ (b) $x^2+y^2-4x-5y=0$

(b) $x^2+y^2+4x+5y=0$ (d) N.O.T.



Sol- Circle intercept x-axis

$2\sqrt{g^2-c} = 4$ $c=0$

$\cancel{2}\sqrt{g^2} = \cancel{4}\,2$

$\sqrt{g^2} = 2$

$g^2 = 4$

$g = \pm\sqrt{4} = \pm 2$

$2\sqrt{f^2-c} = 5$

$2\sqrt{f^2-c} = \frac{5}{2}$

$f^2 = \pm\left(\frac{5}{2}\right)^2 = \pm\frac{25}{4}$

$f = \pm\frac{5}{2}$

eq^n of circle $x^2+y^2+2gx+2fy+c=0$

$x^2+y^2-4x-5y=0$ Ans

Ques Circle of radius 2 lies in the Ist quadrant and touch both the axis of coordinate the equation of circle with centre (6,5) and touching the above circle externally is

(a) $x^2+y^2+12x-10y+52=0$ (b) $x^2+y^2-12x+10y+52=0$

(c) $x^2+y^2-12x-10y+52=0$ (d) N.O.T.

Sol If circle touch both axis

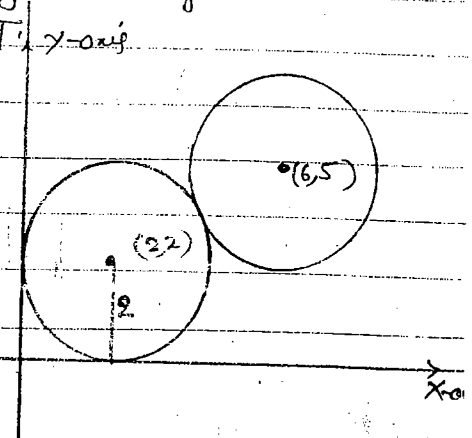
$h=k=r$

$h=k=2$

Centre $=(2,2)$

radius $=2$

And Given that $C_2=(6,5)$



Since circle touches externally

$$C_1C_2 = r_1 + r_2$$

$$\sqrt{(6-2)^2+(5-2)^2} = 2+r_2$$

$$\sqrt{4^2+3^2} = 2+r_2$$

$$\sqrt{25} = 2+r_2$$

$$5 = 2+r_2 \Rightarrow r_2 = 5-2 = 3$$

Hence eqn of circle

$$(x-6)^2+(y-5)^2 = 3^2$$

$$x^2+y^2-12x-10y+36+25-9=0$$

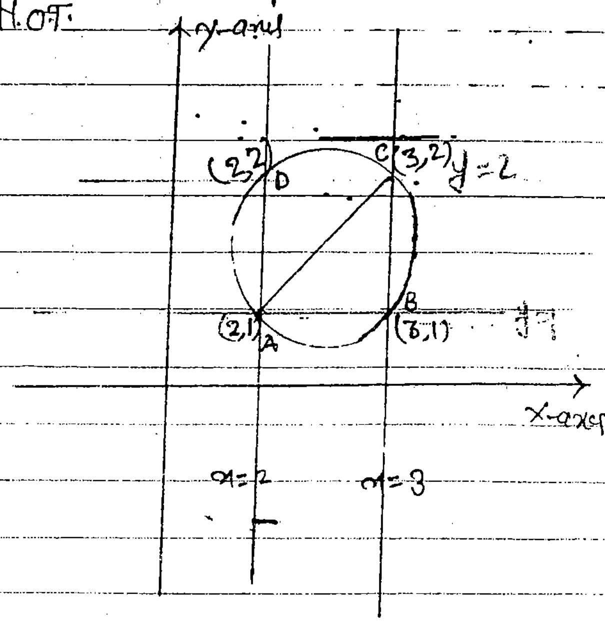
$$x^2+y^2-12x-10y+52=0 \quad \text{Ans}$$

Ques – The sides of square $x=2$ $x=3$ $y=1$ $y=2$. The eqn of circle drawn on diagonal of the square at its diameter is

(a) $x^2+y^2-5x-3y+8=0$ ✓ (b) $x^2+y^2+5x-3y+8=0$

(c) $x^2+y^2+5x+8y+8=0$ (d) N.O.T.

Sol eqn of diagonal of circle



$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$$(x-2)(x-3)+(y-1)(y-2)=0$$

$$x^2-3x-2x+6+y^2-2y-y+2=0$$

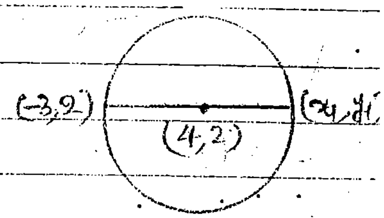
$$x^2+y^2-5x-3y+8=0 \quad \text{Ans}$$

Ques If the Coordinate of the one ends of diameter of the Circle $x^2+y^2-8x-4y+c=0$ are $(-3,2)$ then the Coordinat at the other ends are.

(a) $(5,3)$ (b) $(6,2)$ (c) $(1,-8)$ (d) $(11,2)$

Sol eq^n of Circle

$$x^2+y^2-8x-4y+c=0$$



$$g=-4 \quad f=-2$$

Centre $=(-g,-f)=(4,2)$

Let (x_1,y_1) be the other ends of diameter

$$\frac{-3+x_1}{2}=4 \Rightarrow -3+x_1=8 \Rightarrow x_1=11$$

and $$\frac{2+y_1}{2}=2 \Rightarrow 2+y_1=4 \Rightarrow y_1=2$$

$$(x_1,y_1)\to(11,2)$$

Ques - The obcesa of two points A and B are the roots of eq^n $x^2+2ax+b^2=0$ and the Coordinate are the roots of eq^n $y^2+2by+q^2=0$. then the eq^n of Circle with AB as diameter

Sol eq^n

$$x^2+2ax+b^2 \quad \text{---} (1)$$

let x_1 x_2 be roots

$$x_1+x_2=-2a$$
$$x_1x_2=b^2$$

eq^n

$$y^2+2by+q^2$$

let y_1, y_2 be the roots

$$y_1+y_2=-2b$$
$$y_1y_2=q^2$$

Eq^n of diameter

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$$\{x^2-(x_1+x_2)x+x_1x_2\}+\{y^2-(y_1+y_2)y+y_1y_2\}=0 \quad \text{---} (A)$$

putting the roots value in (A)

$(x^2+2ax+b^2)+(y^2+2by+q^2)=0$

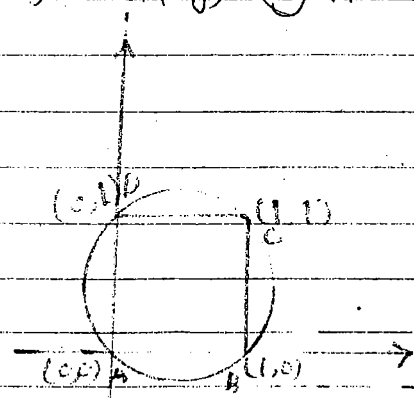
$x^2+y^2+2ax+2by+b^2+q^2=0$ Ans

Ques ABCD is a Square whose length of side is 1. If AB & AD be taken as axis then the circle circumscribing the square will have the eq^n

(a) $x^2+y^2=1(x+y)$ (b) $x^2+y^2=(x-y)$ (c) $(x^2+y^2=1(x+2y)$ (d) NOT.

Sol Let eq^n of General circl

$x^2+y^2+2gx+2fy+c=0$ —— (A)



It passes $(0,0)$

$c=0$

If passes $(1,0)$ $2g=-1$

$g=-\frac{1}{2}$

If passes through $(0,1)$

$f=-\frac{1}{2}$

if $(1,1)$

$1+1+2g+2f=0$

$\cancel{2}(g+f)=-\cancel{2}$ $\Rightarrow g+f=-1$

Hence eq^n of circle becomes

$x^2+y^2+\cancel{2}\cdot(-\frac{1}{\cancel{2}})x+\cancel{2}\cdot(-\frac{1}{\cancel{2}})y+0=0$

$x^2+y^2-x-y=0$

$x^2+y^2=1(x+y)$ Ans

Ques A circle of radius 5 touches the coordinate axis in the Ist quadrant. If the circle way one complete rowlo on x-axis along positive direction of x-axis then its eq^n in the new position.

Functions

(*) Set :- Set is a well defined Collection of objects.

Let $A = \{1, 2, 3, 4\}$

and $B = \{5, 6, 7, 8\}$

then $A \times B = \{(1,5), (1,6), (1,7) (1,8) (2,5) (2,6) (2,7) (2,8)$
$(3,5), (3,6) (3,7) (3,8), (4,5), (4,6) (4,7) (4,8)\}$

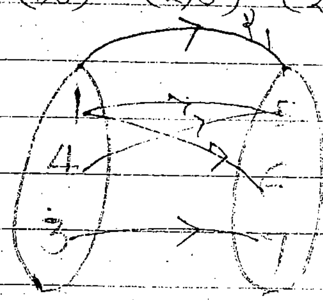
then $A \times B$ is called Cartesian Product of two Sets

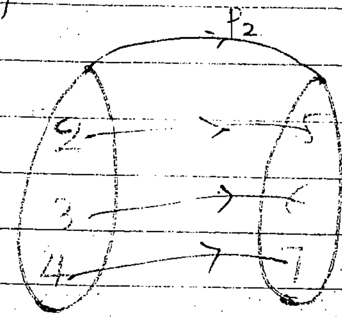
Consider Subset of $(A \times B)$

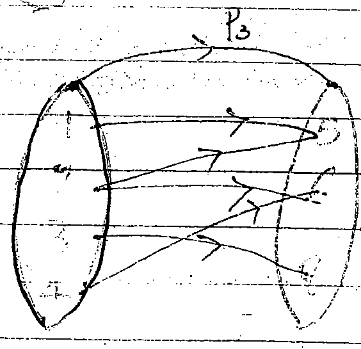
$P_1 = \{(1,5), (1,6), (4,5), (8,7)\}$

$P_2 = \{(2,5) (3,6) (4,7)\}$

$P_3 = \{(1,5) (2,5), (2,6) (3,8), (4,6)\}$

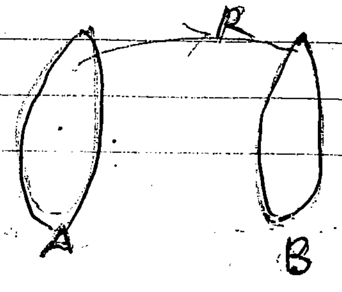






Then every Subset of $(A \times B)$ will give us a relation.
i.e

A relation $R : A \longrightarrow B$ is Subset of $A \times B$



A relation $R: A \longrightarrow B$ satisfying following conditions.

⟨1⟩ Each element of A should be related by some element of B.

⟨2⟩ More than one element of A can be related with ~~sigle~~ single element of B but one element of A can not be related with more than one elements of B is known as mapping.

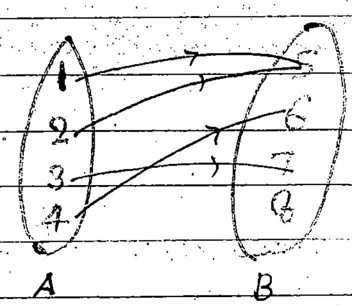
i.e. "Every mapping is a relation but every relation is not a mapping."

Here P_1 and P_3 are not mapping as in P_1 1 is related to 5, 6 and in P_3 7 is related with 5 and 6.

But P_2 is a mapping

Here element of B are known as images of some element of A.

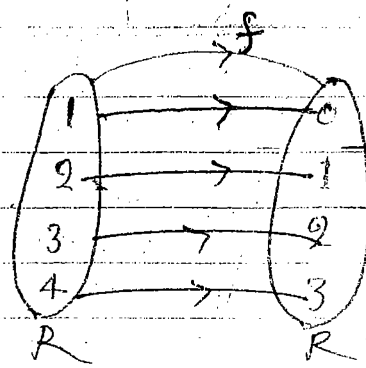
and element of A are known as Pre-images of elements of B.



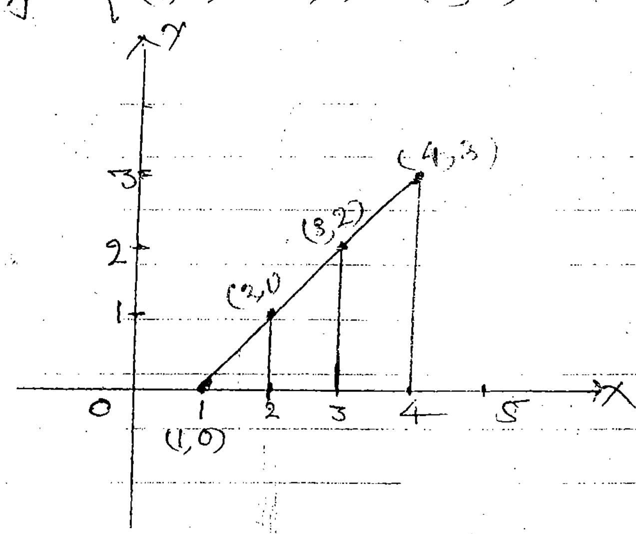
Also A and B are known as domain and Co-domain respectivily –

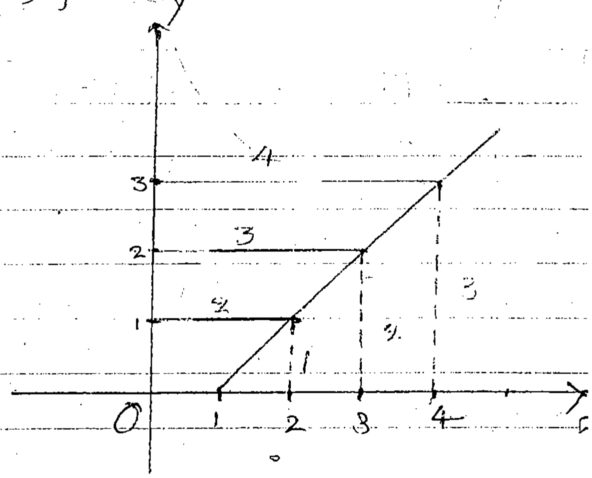
If $A = B = R$

then $f: A \longrightarrow B$ is $f: R \longrightarrow R$ is known as real function.



$f = \{(1,0)\ (2,1)\ (3,2)\ (4,3)\}$





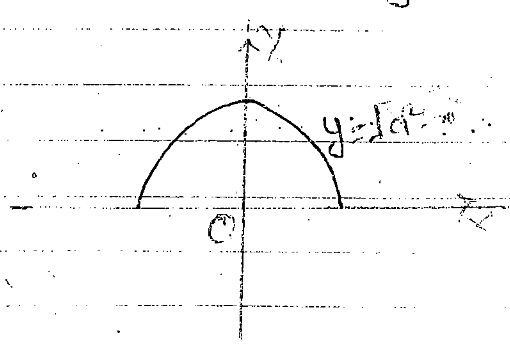
Note—

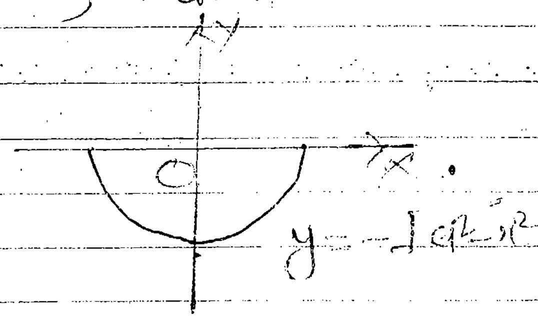
1.) Image पाने के लिए y-axis के Parallel.

2.) Pre-image पाने के लिए x-axis के parallel.

1- Circle:- $x^2+y^2=a^2 \Rightarrow y^2=a^2-x^2 \Rightarrow y=\pm\sqrt{a^2-x^2}$

$y=\sqrt{a^2-x^2},\ -\sqrt{a^2-x^2}$

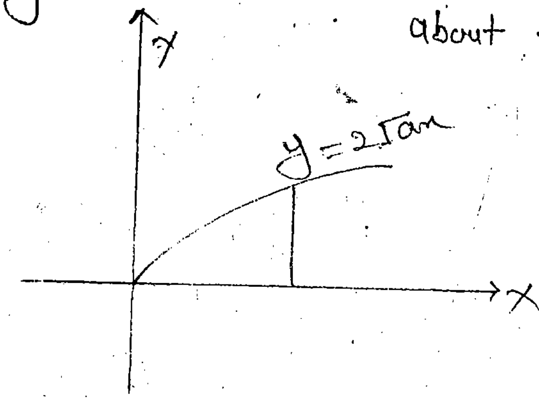


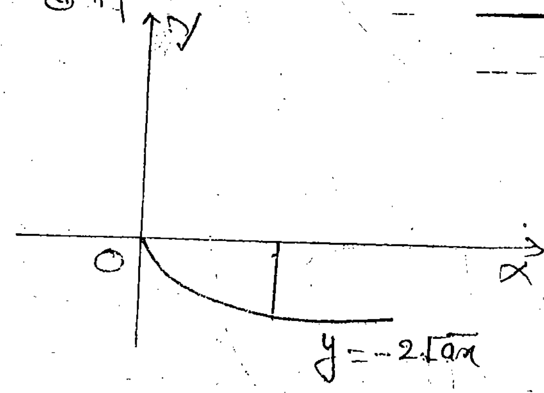


2) Parabola:-
$y^2 = 4ax$

यदि y मे Second degree है तो x-axis के about symmetric होगा।

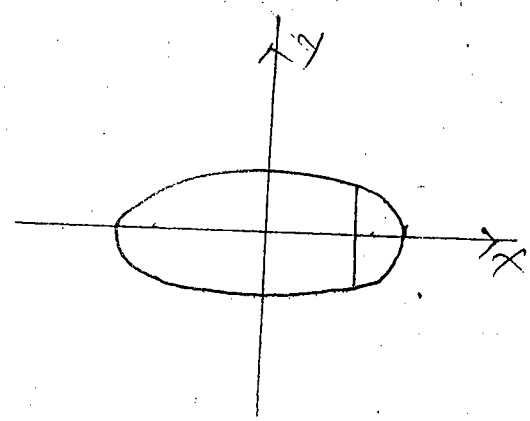
$y = \pm 2\sqrt{ax}$ यदि x मे second degree है तो y-axis के about symmetric होगा।





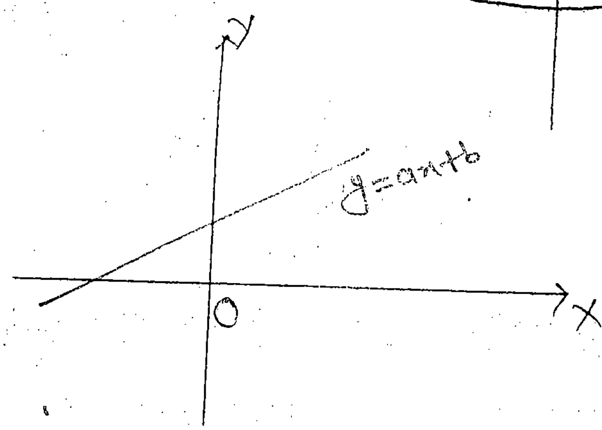
3> **Ellipse :-** eqⁿ of ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



4> **Straight line-**

$$y = ax+b$$



"A graphical approach we note that every function is a curve" but every curve is not a function."

i.e. If any line parallel to y-axis cuts the graph more than one point then such curve will not be a function.

i.e. A line parallel to y-axis should not to cut the curve at one and only one point

Example- Circle $x^2+y^2=a^2$

$y^2=4ax$, $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x=k$ are not a function but $y=\sin x$. $y=2\sqrt{ax}$ $y=ax+b$ etc are function.

घात → सम – समान चिन्ह , विषम → विपरीत

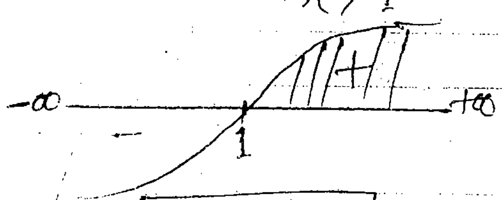
Method of Interval :

$>$ → R.H.S

$<$ → L.H.S

(1) Solve $x-1>0$

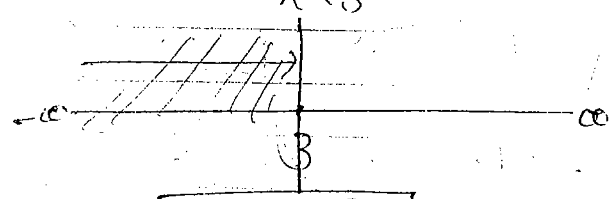
$x>1$



$\boxed{x\in(1,\infty)}$ Ans

(2) Solve $x-3<0$

$x<3$



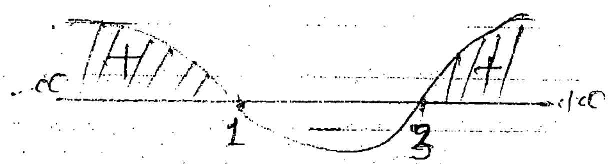
$\boxed{x\in(-\infty,3)}$ Ans

(3) Solve $(x-1)(x-3)>0$

$(x-1)\ (x-3)>0$

$x-1=0 \qquad x-3=0$

$x=1 \qquad x=3$

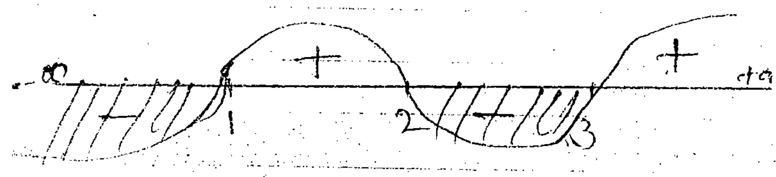


$\boxed{x\in(-\infty,1)\cup(3,\infty)}$

(4) Solve $\dfrac{(x-1)(x-2)}{(x-3)}<0$

$\dfrac{(x-1)\ (x-2)}{(x-3)}<0$

$x=1 \quad x=2 \quad x=3$

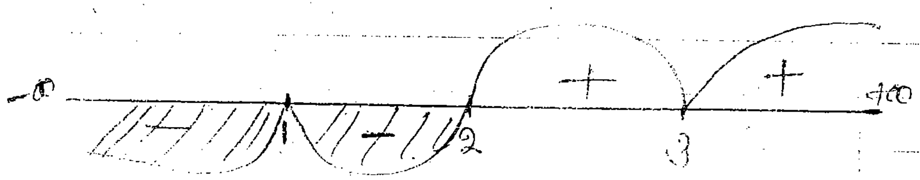


Since $x<0$ (negative sign)

$\boxed{x\in(-\infty,1)\cup(2,3)}$

(5) Solve $(x-1)^2(x-2)(x-3)^4<0$

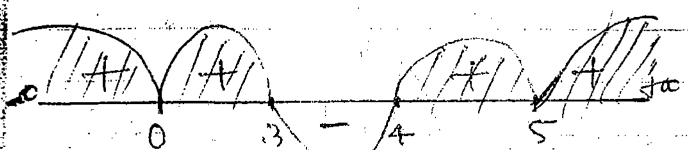
$(x-1)^2(x-2)(x-3)^4<0$



$\boxed{x\in(-\infty,1)\cup(1,2)}$

(6) Solve $\dfrac{x^2(x-3)}{(x-4)(x-5)^2} > 0$

$x=0,\ x=3,\ x=4,\ x=5$



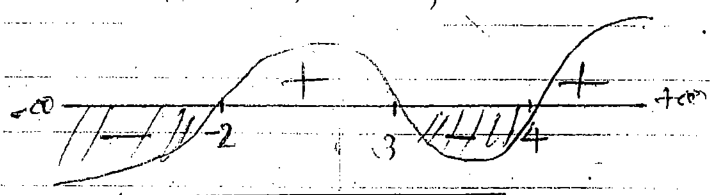
$$\boxed{x \in (-\infty,0)\cup(0,3)\cup(4,5)\cup(5,\infty)}$$

(7) Solve $\dfrac{(x+2)(3-x)}{(x-4)} > 0$

$$\frac{(x+2)\ (\cancel{3}-x)}{(x-4)} > 0$$

$$\frac{(x+2)(x-3)}{(x-4)} < 0$$

$x=-2,\ x=3,\ x=4$



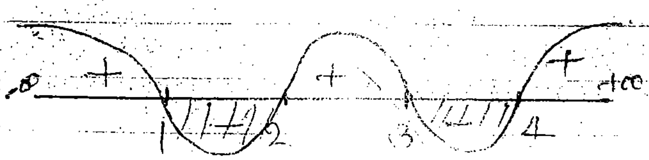
$$\boxed{x \in (-\infty,2)\cup(3,4)}$$

(8) Solve $\dfrac{x^2-4x+3}{x^2-6x+8} < 0$

$$\frac{x^2-4x+3}{x^2-6x+8} < 0$$

$$\frac{(x-1)(x-3)}{(x-2)(x-4)} < 0$$

$x=1\quad x=3,\ x=2,\ x=4$



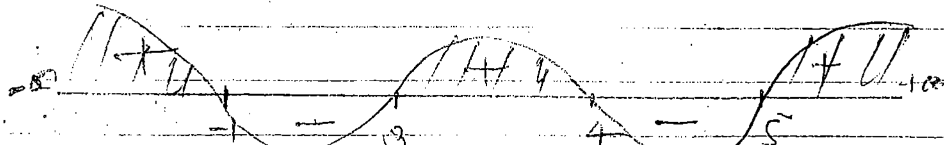
$$\boxed{x \in (1,2)\cup(3,4)}$$

(9) Solve $\dfrac{x^2-2x-3}{x^2-9x+20} > 0$

$$\frac{x^2-2x+3}{x^2-9x+20} > 0$$

$$\frac{(x+1)(x-3)}{(x-4)(x-5)} > 0$$

$x=-1,\ x=3,\ x=4,\ x=5$



$$\boxed{x \in (-\infty,-1)\cup(3,4)\cup(5,\infty)}$$